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Two Interacting Strike Slip Faults in a Viscoelastic Half Space Under Increasing Tectonic Forces

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Abstract: Two interacting inclined strike slip faults are considered in a viscoelastic half space under the action of tectonic forces which increases with time. Tectonic forces generated due to mantle convection and other related phenomena have been the main driving forces for the movement of Lithospheric plates leading to earthquake. It may be noted that during the aseismic period in between two major seismic events, stresses built up gradually due to the action of tectonic forces. It is assumed that the accumulated stress when exceeds a threshold value, a creeping movement across the fault sets in. Analytical expressions for displacement, stresses and strain are being obtained using suitable mathematical techniques, both before and after the fault movement. It is expected that the numerical computation will give us an idea on the rate of stress accumulation in the media under such conditions.

Keywords: Aseismic period, Correspondence principle, Creeping movement, Mantle convection, Strike slip fault, Tectonic forces, Viscoelastic half space.

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1. INTRODUCTION

Tectonic forces generated due to mantle convection and other related phenomena plays important role in the nature of stress accumulation during the aseismic period in seismically active regions. The tectonic forces has been assumed to be of the form $\tau_\infty(t) = \tau_\infty(0)(1+\mathrm{kt})$, where k is a constant, whose value depends upon the model parameters. Interacting effect among the neighbouring faults have been considered by [1] , assuming $\tau_\infty(t)$ to be a constant, independent of time. In the present paper the effect due to the increasing value of $\tau_\infty(t)$ will be investigated.

2. FORMULATION

We consider a viscoelastic half space of Maxwell type representing the Lithosphere-asthenosphere system. Two long inclined and buried strike-slip faults F_1 and F_2 are taken to be situated in a half space with inclination θ_1 and θ_2 respectively with the horizontal as shown in Fig. 1.

Let d_1 and $\mathrm{d}_2$are the depths of the upper edges of the faults below the free surface and D is the distance measured horizontally between the upper edges of the faults. θ_1 and θ_2 are the inclination of the faults with the horizontal. D_1 and D_2 are the lengths of the faults F_1 and F_2 respectively.

We introduce a system of rectangular Cartesian coordinate axes $(\mathrm{o}, \mathrm{y}_1, \mathrm{y}_2, \mathrm{y}_3)$, $(\mathrm{o}', \mathrm{y}_1', \mathrm{y}_2', \mathrm{y}_3')$, $(\mathrm{o}'', \mathrm{y}_1'', \mathrm{y}_2'', \mathrm{y}_3'')$ as shown in the following figure. The relationship between these coordinate system are given by:

$$\left.\begin{array}{c}
\mathrm{y}'_1 = \mathrm{y}_1 \\
\mathrm{y}'_2 = \mathrm{y}_2\sin\theta_1 - (\mathrm{y}_3 - \mathrm{d}_1)\cos\theta_1 \\
\mathrm{y}'_3 = \mathrm{y}_2\cos\theta_1 + (\mathrm{y}_3 - \mathrm{d}_1)\sin\theta_1 \\
\text{and} \\
\mathrm{y}''_1 = \mathrm{z}_1 \\
\mathrm{y}''_2 = \mathrm{z}_2\sin\theta_2 - \mathrm{z}_3\cos\theta_2 \\
\mathrm{y}''_3 = \mathrm{z}_2\cos\theta_2 + \mathrm{z}_3\sin\theta_2
\end{array}\right\}$$

where $\mathrm{z}_2 = \mathrm{y}_2 - \mathrm{D}, \quad \mathrm{z}_3 = \mathrm{y}_3 - \mathrm{d}_2$.

The lengths of the faults are assumed to be large enough compare to their widths so that choosing $\mathrm{y}_1$axes along the strike of the fault F_1, the displacement stresses and strain become independent of y_1. We thus have a two dimensional problem where the displacement, stresses and strain are functions of $\mathrm{y}_2, \mathrm{y}_3$ and of t.

2.1 Constitutive Equations

The constitutive Equations have been taken as:

$$\left.\begin{array}{l}
\left(\frac{1}{\eta} + \frac{1}{\mu}\frac{\partial}{\partial t}\right)\tau_{12} = \frac{\partial}{\partial t}(e_{12}) = \frac{\partial^2 u}{\partial t \partial y_2} \\
\left(\frac{1}{\eta} + \frac{1}{\mu}\frac{\partial}{\partial t}\right)\tau_{13} = \frac{\partial}{\partial t}(e_{13}) = \frac{\partial^2 u}{\partial t \partial y_3}
\end{array}\right\} \qquad (1)$$

where η is the effective viscosity and μ is the effective rigidity of the material.

2.2. Stress equation of motion

The stresses satisfy the following equation of motion:

$$\left.\begin{array}{c}
\frac{\partial}{\partial y_2}(\tau_{12}) + \frac{\partial}{\partial y_3}(\tau_{13}) = 0 \\
(-\infty < y_2 < \infty, \ \ y_3 \geq 0, \ \ t \geq 0)
\end{array}\right\} \qquad (2)$$

[Assuming that the external forces do not change significantly during our investigation and neglecting the inertial term which is very small during the aseismic period]

2.3 Initial conditions

Let $(u)_0, (\tau_{12})_0, (\tau_{13})_0$ and $(e_{12})_0$ are the values of u, τ_{12}, τ_{13}, e_{12} respectively at time t=0. [t=0 representing an instant when the model is in aseismic state]

2.3. Boundary conditions

The boundary conditions are:

$$\left.\begin{array}{l}
\tau_{13} = 0 \text{ on } y_3 = 0\,, (-\infty < y_2 < \infty, \quad t \geq 0) \\
\tau_{13} \to 0 \text{ as } y_3 \to \infty\,, (-\infty < y_2 < \infty, \quad t \geq 0)
\end{array}\right\} \qquad (3)$$

Mantle convection introduces tectonic forces in the lithosphere-asthenosphere system far away from the faults which causes the faults to slip leading to an earthquake. We represent these tectonic forces by $\tau_\infty(t)$ and assume it to be a slowly increasing function of time and write

$\tau_\infty(t) = \tau_\infty(0)(1+kt)$, where $k > 0$,

Then, the relevant boundary conditions become:

$$\left.\begin{array}{l}
\tau_{12} \to \tau_\infty(t) = \tau_\infty(0)(1+kt), (k > 0) \\
\quad \text{as } |y_2| \to \infty, \text{for } y_3 \geq 0, t \geq 0.
\end{array}\right\} \qquad (4)$$

$$\left.\begin{array}{l}
\tau_\infty(0) = \text{The value of } \tau_\infty(t) \text{ at t} = 0. \\
\tau_{12}(0) \to \tau_\infty(0) \text{ as } |y_2| \to \infty, \text{for t} = 0.
\end{array}\right\} \qquad (5)$$

Now differentiating partially equation (1) with respect to y_2 and with respect to y_3 and adding them using equation (2) we get,

$$\nabla^2 u(y_2, y_3, t) = 0 \qquad (6)$$

3. SOLUTION FOR DISPLACEMENTS, STRESSES AND STRAINS IN THE ABSENCE OF ANY FAULT MOVEMENT

In the absence of any fault movement the displacement and stresses are continuous through out the system. Stress accumulated due to the action of τ_∞ . In order to obtain the expressions for displacement, strain and stresses we take Laplace transform of (1) to (6) with respect to t. The resulting boundary value problem can be solved by taking integral transforms of the constitutive equations and the boundary conditions with respect to t. The solutions are given below.

$$\left.\begin{array}{c}
u = (u)_0 + y_2\tau_\infty(0)\left[\frac{kt}{\mu} + \frac{t}{\eta} + \frac{kt^2}{2\eta}\right] \\
e_{12} = (e_{12})_0 + \tau_\infty(0)\left[\frac{kt}{\mu} + \frac{t}{\eta} + \frac{kt^2}{2\eta}\right] \\
\tau_{12} = (\tau_{12})_0 e^{-\frac{\mu t}{\eta}} + \tau_\infty(0)\left(1 + kt - e^{-\frac{\mu t}{\eta}}\right) \\
\tau_{13} = (\tau_{13})_0 e^{-\frac{\mu t}{\eta}} \\
\tau_{1'2'} = \text{ The stresses across the fault } F_1 \\
= \tau_{12}\sin\theta_1 - \tau_{13}\cos\theta_1 \\
= (\tau_{1'2'})_0 e^{-\frac{\mu t}{\eta}} + \tau_\infty(0)\left(1 + kt - e^{-\frac{\mu t}{\eta}}\right)\sin\theta_1, t \geq 0 \\
\tau_{1''2''} = \text{ The stresses across the fault } F_2 \\
= \tau_{12}\sin\theta_2 - \tau_{13}\cos\theta_2 \\
= (\tau_{1''2''})_0 e^{-\frac{\mu t}{\eta}} + \tau_\infty(0)\left(1 + kt - e^{-\frac{\mu t}{\eta}}\right)\sin\theta_2, t \geq 0
\end{array}\right\} \qquad (7)$$

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From the above result we find that $\tau_{1'2'}$ and $\tau_{1''2''}$ are increasing with time as

$\tau_\infty(0)\left(1+kt-e^{-\frac{\mu t}{\eta}}\right)sin\theta_1$ and $\tau_\infty(0)\left(1+kt-e^{-\frac{\mu t}{\eta}}\right)sin\theta_2$ respectively, noting that $e^{-\frac{\mu t}{\eta}} \to 0$ as $t \to \infty$.

This has been shown in Fig. 2 for before fault movement ,different values of k.

We now assume that the fault F_1 and F_2, and the local rheological condition near them are such that they can withstand a stress of magnitude $(\tau_c)_1 = 200$ bar, $(\tau_c)_2 = 250$ bar respectively. We further assume $\tau_\infty(0) = 50$ bar. It is found that the time taken to reach these critical values are $\mathrm{T}_1 = 117$ years and $\mathrm{T}_2 = 152$ years (assuming $\theta_1 = \theta_2 = 60^0$).

We consider first when the stress $\tau_{1'2'}$ reaches the value $(\tau_c)_1$ after T_1 years and the fault F_1 starts creeping with a velocity v_1 cm/year. This creeping dislocation may be characterised by

$$[u]_{F_1} = U_1(t_1)f(y'_3)H(T-t_1) \tag{8}$$

4. SOLUTION OF THE PROBLEM AFTER THE CREEPING MOVEMENT ACROSS F_1 FOR $0 < t \le T_1 < T_2$

After the commencement of the fault creep across F_1 the boundary value problem for displacement, strain and stresses can be written as:

4.1 Constitutive Equations: As in (1).

4.2 Stress equation of motion: As in (2). [Neglecting the inertial terms]

4.3 Boundary conditions: As in (3), (4) and (5).

In addition there is a dislocation condition given by

$$[u]_{F_1} = U_1(t_1)f(y'_3)H(T-t_1)$$

We seek to obtain the solutions for the following forms:

$$\left.\begin{aligned} u &= (u)_1 + (u)_2 \\ e_{12} &= (e_{12})_1 + (e_{12})_2 \\ \tau_{12} &= (\tau_{12})_1 + (\tau_{12})_2 \\ \tau_{13} &= (\tau_{13})_1 + (\tau_{13})_2 \\ \tau_{1'2'} &= (\tau_{1'2'})_1 + (\tau_{1'2'})_2 \\ \tau_{1''2''} &= (\tau_{1''2''})_1 + (\tau_{1''2''})_2 \end{aligned}\right\} \tag{9}$$

where $(u)_1$, $(e_{12})_1$, $(\tau_{12})_1$, $(\tau_{13})_1$, $(\tau_{1'2'})_1$ and $(\tau_{1''2''})_1$ satisfy the equations (1), (2), (3), (4), (5) and $(u)_2$, $(e_{12})_2$, $(\tau_{12})_2$, $(\tau_{13})_2$, $(\tau_{1'2'})_2$ and $(\tau_{1''2''})_2$ satisfied equations (1), (2), (3), (4), (5) with replacing the equation $\tau_{12} \to \tau_\infty(t)$ by

$$\tau_{12} \to 0 \;\; as \;\; |y_2| \to \infty, for \;\; y_3 \ge 0, \; t \ge 0. \tag{10}$$

and an additional dislocation condition

$$[u]_{F_1} = U_1(t_1)f(y'_3)H(t-T_1)$$

where [u] is the discontinuity in u across F_1 and $H(t-T_1)$ is Heaviside unit step function.

The solutions for $(u)_1$, $(e_{12})_1$, $(\tau_{12})_1$, $(\tau_{13})_1$, $(\tau_{1'2'})_1$ and $(\tau_{1''2''})_1$ can be obtained as in the case when there was no fault movement so that

$$u = (u)_0 + y_2\tau_\infty(0)\left[\frac{kt}{\mu} + \frac{t}{\eta} + \frac{kt^2}{2\eta}\right]$$

$$e_{12} = (e_{12})_0 + \tau_\infty(0)\left[\frac{kt}{\mu} + \frac{t}{\eta} + \frac{kt^2}{2\eta}\right]$$

$$\tau_{12} = (\tau_{12})_0 e^{-\frac{\mu t}{\eta}} + \tau_\infty(0)\left(1+kt-e^{-\frac{\mu t}{\eta}}\right)$$

$$\tau_{13} = (\tau_{13})_0 e^{-\frac{\mu t}{\eta}}$$

$$\begin{aligned} \tau_{1'2'} &= \text{The stresses across the fault } F_1 \\ &= \tau_{12}sin\theta_1 - \tau_{13}cos\theta_1 \\ &= (\tau_{1'2'})_0 e^{-\frac{\mu t}{\eta}} + \tau_\infty(0)\left(1+kt-e^{-\frac{\mu t}{\eta}}\right)sin\theta_1, t \ge 0 \end{aligned}$$

$$\begin{aligned} \tau_{1''2''} &= \text{The stresses across the fault } F_2 \\ &= \tau_{12}sin\theta_2 - \tau_{13}cos\theta_2 \\ &= (\tau_{1''2''})_0 e^{-\frac{\mu t}{\eta}} + \tau_\infty(0)\left(1+kt-e^{-\frac{\mu t}{\eta}}\right)sin\theta_2, t \ge 0 \end{aligned}$$

For finding the solutions for $(u)_2$, $(e_{12})_2$, $(\tau_{12})_2$, $(\tau_{13})_2$, $(\tau_{1'2'})_2$ and $(\tau_{1''2''})_2$ we use a modified Green's function method developed by Maruyama (1966)[2], Rybicki(1871)[3], correspondence principle and integral transforms.

The boundary value problem for $(u)_2$, $(e_{12})_2$, $(\tau_{12})_2$, $(\tau_{13})_2$ which are function of $\mathrm{y}_2, \mathrm{y}_3$ and t which satisfies (1) to (6), (8), (10).

$$\left.\begin{aligned} \left(\frac{1}{\eta} + \frac{1}{\mu}\frac{\partial}{\partial t_1}\right)(\tau_{12})_2 &= \frac{\partial^2 (u)_2}{\partial t_1 \partial y_2} \\ \left(\frac{1}{\eta} + \frac{1}{\mu}\frac{\partial}{\partial t_1}\right)(\tau_{13})_2 &= \frac{\partial^2 (u)_2}{\partial t_1 \partial y_3} \end{aligned}\right\} \tag{11}$$

Where, $t_1 = t - T_1 \; (-\infty < y_2 < \infty, \; y_3 \ge 0, \;\; t \ge T_1)$

$$\left.\begin{aligned} \frac{\partial}{\partial y_2}(\tau_{12})_2 + \frac{\partial}{\partial y_3}(\tau_{13})_2 = 0 \\ (-\infty < y_2 < \infty, \;\; y_3 \ge 0, \;\; t \ge T_1) \end{aligned}\right\} \tag{12}$$

and boundary conditions

$$\left.\begin{aligned} (\tau_{13})_2 = 0 \text{ on } y_3 = 0, (-\infty < y_2 < \infty, \;\; t \ge T_1) \\ (\tau_{13})_2 \to 0 \text{ as } y_3 \to \infty, (-\infty < y_2 < \infty, \;\; t \ge T_1) \\ (\tau_{12})_2 \to 0 \text{ as } |y_2| \to \infty, \; y_3 \ge 0, \;\; t \ge T_1 \end{aligned}\right\} \tag{13}$$

$$(\nabla^2 u)_2 = 0 \tag{14}$$

$$[u]_2 = U_1(t_1)f(y'_3)H(t-T_1) \tag{15}$$

across F_1 $(y_2{}' = 0, \;\; 0 \le y'_3 \le \mathrm{D})$ with $U_1(t_1) = 0$ for $t_1 \le 0$

also $(u)_2$, $(e_{12})_2$, $(\tau_{12})_2$, $(\tau_{13})_2 = 0$ for $t_1 \le 0$

To obtain the solutions for $(u)_2$, $(e_{12})_2$, $(\tau_{12})_2$, $(\tau_{13})_2$ we take Laplace transform of (11) to (15) with respect to $t_1 (= t - T_1)$ and obtain a boundary value problem involving $(\bar{u})_2$, $(\overline{e_{12}})_2$, $(\overline{\tau_{12}})_2$, $(\overline{\tau_{13}})_2$ which are the Laplace transform of $(u)_2$, $(e_{12})_2$, $(\tau_{12})_2$, $(\tau_{13})_2$ respectively with respect to t_1 and are defined as

$$\{(\bar{u})_2, (\bar{u})_3, (\overline{\tau_{12}})_2\} = \int_0^\infty \{(u)_2, (u)_3, (\tau_{12})_2\} \; e^{-pt_1} \; dt_1$$

where p is the Laplace transform variable.

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The resulting boundary value problem is characterised by the following relations

$$\left.\begin{aligned}(\overline{\tau_{12}})_2 &= \frac{p}{\left(\frac{p}{\mu}+\frac{1}{\eta}\right)}\frac{\partial(\bar{u})_2}{\partial y_2}\\ (\overline{\tau_{13}})_2 &= \frac{p}{\left(\frac{p}{\mu}+\frac{1}{\eta}\right)}\frac{\partial(\bar{u})_2}{\partial y_3}\\ (-\infty < y_2 < \infty,\ & y_3 \geq 0)\end{aligned}\right\} \tag{16}$$

$$\frac{\partial}{\partial y_2}(\overline{\tau_{12}})_2 + \frac{\partial}{\partial y_3}(\overline{\tau_{13}})_2 = 0 \tag{17}$$

$$\left.\begin{aligned}&(\overline{\tau_{13}})_2 = 0 \text{ on } y_3 = 0,\ (-\infty < y_2 < \infty,\ t \geq 0)\\ &(\overline{\tau_{13}})_2 \to 0 \text{ as } y_3 \to \infty,\ (-\infty < y_2 < \infty,\ t \geq 0)\\ &(\overline{\tau_{12}})_2 \to 0 \text{ as } |y_2| \to \infty,\ \text{for } y_3 \geq 0\end{aligned}\right\} \tag{18}$$

$$\nabla^2(\bar{u})_2 = 0 \tag{19}$$

And $[(\bar{u})_2] = \overline{U_1}(\mathrm{p})\mathrm{f}\left(y'_3\right)$
across F_1: $y'_2 = 0,\ 0 \leq y'_3 \leq \mathrm{D}$ (20)

where $\overline{U_1}(\mathrm{p})$ is the Laplace transform of $U_1(t_1)$ with respect to t_1 so that

$$\overline{U_1}(\mathrm{p}) = \int_0^\infty U_1(t_1)\ e^{-pt_1}\,dt_1$$

To solve this above boundary value problem, a suitably modified form of Green's function technique, developed by Maruyama (1966)[2] and Rybicki (1971)[3] is used :

$$(\bar{u})_2(\mathrm{Q}) = \int(\bar{u})_2(P)\{G^1{}_{13}(Q,P)d\xi_2 - G^1{}_{12}(Q,P)d\xi_3\} \tag{21}$$

where the integration is taken over the fault F_1 and $Q(y_1,y_2,y_3)$ is the field point in the half space, not on the fault, and $P(\xi_1,\xi_2,\xi_3)$ is any point on the fault F_1 and $(\bar{u})_2(P)$ is the discontinuity in $(\bar{u})_2$ across F_1 at the point P while $G^1{}_{13}(Q,P)$ and $G^1{}_{12}(Q,P)$ are two Green's functions are given by :

$$G^1{}_{13}(Q,P) = \frac{1}{2\pi}\left[\frac{y_3-\xi_3}{L^2} - \frac{y_3+\xi_3}{M^2}\right]$$

and

$$G^1{}_{12}(Q,P) = \frac{1}{2\pi}\left[\frac{y_2-\xi_2}{L^2} + \frac{y_2-\xi_2}{M^2}\right]$$

where,
$L^2 = (y_2-\xi_2)^2 + (y_3-\xi_3)^2,\quad M^2 = (y_2-\xi_2)^2 + (y_3+\xi_3)^2$

Now $P(\xi_1,\xi_2,\xi_3)$ being a point on F_1, $0 \leq \xi_2 \leq D_1 cos\theta_1$, $0 \leq \xi_3 \leq D_1 sin\theta_1$ and $\xi_2 = \xi_3 cot\theta_1$. We introduce a change of coordinate axes from (ξ_1,ξ_2,ξ_3) to (ξ'_1,ξ'_2,ξ'_3) connected by the relations

$$\xi_1 = \xi'_1$$
$$\xi_2 = \xi'_2 sin\theta_1 + \xi'_3 cos\theta_1$$
$$\xi_3 = d_1 - \xi'_2 cos\theta_1 + \xi'_3 sin\theta_1$$

so that, $\xi'_2 = 0,\ 0 \leq \xi'_3 \leq D_1$ on F_1 .
Then from (20) using (21) we have

$$(\bar{u})_2(Q) = \frac{\overline{U_1}(\mathrm{p})}{2\pi}\int_0^{D_1} \mathrm{f}\left(\xi'_3\right)\left[\frac{y_2 sin\theta_1 - (y_3-d_1)cos\theta_1}{\xi'^2_3 - 2\xi'_3\{y_2cos\theta_1 + (y_3-d_1)sin\theta_1\} + y_2^2 + (y_3-d_1)^2} + \frac{y_2 sin\theta_1 + (y_3+d_1)cos\theta_1}{\xi'^2_3 - 2\xi'_3\{y_2cos\theta_1 - (y_3+d_1)sin\theta_1\} + y_2^2 + (y_3+d_1)^2}\right]d\xi'_3$$

or,

$$(\bar{u})_2(Q) = \frac{\overline{U_1}(\mathrm{p})}{2\pi}\psi_1(y_2,y_3)$$

where,

$$\psi_1(y_2,y_3) = \int_0^{D_1} \mathrm{f}\left(\xi'_3\right)\left[\frac{y_2 sin\theta_1 - (y_3-d_1)cos\theta_1}{\xi'^2_3 - 2\xi'_3\{y_2cos\theta_1 + (y_3-d_1)sin\theta_1\} + y_2^2 + (y_3-d_1)^2} + \frac{y_2 sin\theta_1 + (y_3+\mathrm{d}_1)cos\theta_1}{\xi'^2_3 - 2\xi'_3\{y_2cos\theta_1 - (y_3+d_1)sin\theta_1\} + y_2^2 + (y_3+d_1)^2}\right]d\xi'_3$$

On taking inverse Laplace transform with respect to t_1 and noting that $(u)_2 = 0$ for $t_1 \leq 0$

$$(u)_2 = \frac{U_1(t_1)}{2\pi}\psi_1(y_2,y_3)H(t-T_1)$$

Now from (16),

$$(\overline{\tau_{12}})_2 = \frac{p}{\left(\frac{p}{\mu}+\frac{1}{\eta}\right)}\frac{\partial(\bar{u})_2}{\partial y_2} = \frac{p}{\left(\frac{p}{\mu}+\frac{1}{\eta}\right)}\frac{U_1(\mathrm{p})}{2\pi}\psi_2(y_2,y_3)$$

where ,

$$\psi_2(y_2,y_3) = \frac{\partial}{\partial y_2}\{\psi_1(y_2,y_3)\}$$

$$= \int_0^{D_1}\mathrm{f}\left(\xi'_3\right)\left[\frac{\xi'^2_3 sin\theta_1 - 2\xi'_3(y_3-d_1) - \{y_2^2-(y_3-d_1)^2\}sin\theta_1 + 2y_2(y_3-d_1)cos\theta_1}{\left[\xi'^2_3 - 2\xi'_3\{y_2cos\theta_1+(y_3-d_1)sin\theta_1\} + y_2^2+(y_3-d_1)^2\right]^2} + \frac{\xi'^2_3 sin + 2\xi'_3(y_3+d_1) - \{y_2^2-(y_3+d_1)^2\}sin\theta_1 - 2y_2(y_3+d_1)cos\theta_1}{\left[\xi'^2_3 - 2\xi'_3\{y_2cos\theta_1-(y_3+d_1)sin\theta_1\} + y_2^2+(y_3+d_1)^2\right]^2}\right]d\xi'_3$$

Now taking inverse Laplace transformation and noting that $(\tau_{12})_2 = 0$ for $t_1 \leq 0$

$$(\tau_{12})_2 = \frac{\mu}{2\pi}H(t-T_1)\left(U_1(t_1) - \frac{\mu}{\eta}\int_0^{t-T_1} U_1(\tau)e^{\frac{-\mu(t-T_1-\tau)}{\eta}}d\tau\right)\psi_2(y_2,y_3)$$

$$(\tau_{13})_2 = \frac{\mu}{2\pi}H(t-T_1)\left(U_1(t_1) - \frac{\mu}{\eta}\int_0^{t-T_1} U_1(\tau)e^{\frac{-\mu(t-T_1-\tau)}{\eta}}d\tau\right)\psi_3(y_2,y_3)$$

where,
$\psi_3(y_2,y_3)$

$$= -\int_0^{D_1}\mathrm{f}\left(\xi'_3\right)\left[\frac{\xi'^2_3 cos\theta_1 - 2\xi'_3 y_2 + \{y_2^2-(y_3-d_1)^2\}cos\theta_1 + 2y_2(y_3-d_1)sin\theta_1}{\left[\xi'^2_3 - 2\xi'_3\{y_2cos\theta_1+(y_3-d_1)sin\theta_1\} + y_2^2+(y_3-d_1)^2\right]^2} - \frac{\xi'^2_3 cos\theta_1 - 2\xi'_3 y_2 + \{y_2^2-(y_3+d_1)^2\}cos\theta_1 - 2y_2(y_3+d_1)sin\theta_1}{\left[\xi'^2_3 - 2\xi'_3\{y_2cos\theta_1-(y_3+d_1)sin\theta_1\} + y_2^2+(y_3+d_1)^2\right]^2}\right]d\xi'_3$$

We assuming $U_1(t_1) = \mathrm{v_1t_1}$

$$(u)_2 = H(t-T_1)\frac{\mathrm{v_1t_1}}{2\pi}\psi_1(y_2,y_3)$$

$$(e_{12})_2 = H(t-T_1)\frac{\mathrm{v_1t_1}}{2\pi}\psi_2(y_2,y_3)$$

$$(\tau_{12})_2 = H(t-T_1)\frac{\mathrm{v_1}\eta}{2\pi}\left(1 - e^{\frac{-\mu(t-T_1)}{\eta}}\right)\psi_2(y_2,y_3)$$

$$(\tau_{13})_2 = \mathrm{H}(t-T_1)\frac{\mathrm{v_1}\eta}{2\pi}\left(1 - e^{\frac{-\mu(t-T_1)}{\eta}}\right)\psi_3(y_2,y_3)$$

$$\left(\tau_{1'2'}\right)_2 = H(t-T_1)\frac{\mathrm{v_1}\eta}{2\pi}\left(1 - e^{\frac{-\mu(t-T_1)}{\eta}}\right)(\psi_2\ sin\theta_1 - \psi_3\ cos\theta_1)$$

$$\left(\tau_{1''2''}\right)_2 = H(t-T_1)\frac{\mathrm{v_1}\eta}{2\pi}\left(1 - e^{\frac{-\mu(t-T_1)}{\eta}}\right)(\psi_2\ sin\theta_2 - \psi_3\ cos\theta_2)$$

5. SOLUTION FOR THE DISPLACEMENT, STRAIN AND STRESSES AFTER THE COMMENCEMENT OF THE FAULT CREEP ACROSS F_2

When the accumulated stress near F_2 exceeds the threshold value $(\tau_c)_2$ a creeping movement with a velocity v_2 cm/year across F_2 starts. Providing in a similar way the final solutiopn after $t > T_2$ is given by

$$\left.\begin{aligned}
u &= (u)_0 + y_2\tau_\infty(0)\left[\frac{kt}{\mu}+\frac{t}{\eta}+\frac{kt^2}{2\eta}\right] + H(t-T_1)\frac{v_1t_1}{2\pi}\psi_1(y_2,y_3)\\
&\quad +H(t-T_2)\frac{v_2t_2}{2\pi}\phi_1(y_2,y_3)\\
e_{12} &= (e_{12})_0 + \tau_\infty(0)\left[\frac{kt}{\mu}+\frac{t}{\eta}+\frac{kt^2}{2\eta}\right] + H(t-T_1)\frac{v_1t_1}{2\pi}\psi_2(y_2,y_3)\\
&\quad +H(t-T_2)\frac{v_2t_2}{2\pi}\phi_2(y_2,y_3)\\
\tau_{12} &= (\tau_{12})_0e^{-\frac{\mu t}{\eta}} + \tau_\infty(0)\left(1+kt-e^{-\frac{\mu t}{\eta}}\right)\\
&\quad +H(t-T_1)\frac{v_1\eta}{2\pi}\left(1-e^{\frac{-\mu(t-T_1)}{\eta}}\right)\psi_2(y_2,y_3)\\
&\quad +H(t-T_2)\frac{v_2\eta}{2\pi}\left(1-e^{\frac{-\mu(t-T_2)}{\eta}}\right)\phi_2(y_2,y_3)\\
\tau_{13} &= (\tau_{13})_0e^{-\frac{\mu t}{\eta}} + \mathrm{H}(t-T_1)\frac{v_1\eta}{2\pi}\left(1-e^{\frac{-\mu(t-T_1)}{\eta}}\right)\psi_3(y_2,y_3)\\
&\quad +\mathrm{H}(t-T_2)\frac{v_2\eta}{2\pi}\left(1-e^{\frac{-\mu(t-T_2)}{\eta}}\right)\phi_3(y_2,y_3)\\
\tau_{1'2'} &= \left(\tau_{1'2'}\right)_0e^{-\frac{\mu t}{\eta}} + \tau_\infty(0)\left(1+kt-e^{-\frac{\mu t}{\eta}}\right)\sin\theta_1\\
&\quad +H(t-T_1)\frac{v_1\eta}{2\pi}\left(1-e^{\frac{-\mu(t-T_1)}{\eta}}\right)(\psi_2\sin\theta_1-\psi_3\cos\theta_1)\\
&\quad +H(t-T_2)\frac{v_2\eta}{2\pi}\left(1-e^{\frac{-\mu(t-T_2)}{\eta}}\right)(\phi_2\sin\theta_1-\phi_3\cos\theta_1)\\
\tau_{1''2''} &= \left(\tau_{1''2''}\right)_0e^{-\frac{\mu t}{\eta}} + \tau_\infty(0)\left(1+kt-e^{-\frac{\mu t}{\eta}}\right)\sin\theta_2\\
&\quad +H(t-T_1)\frac{v_1\eta}{2\pi}\left(1-e^{\frac{-\mu(t-T_1)}{\eta}}\right)(\psi_2\sin\theta_2-\psi_3\cos\theta_2)\\
&\quad +H(t-T_2)\frac{v_2\eta}{2\pi}\left(1-e^{\frac{-\mu(t-T_2)}{\eta}}\right)(\phi_3\sin\theta_2-\phi_3\cos\theta_2)
\end{aligned}\right\} \tag{22}$$

where,

$$\phi_1(y_2,y_3) = \int_0^{D_2} f_2(\eta'_3)\left[\frac{(y_2-D)\sin\theta_2-(y_3-d_2)\cos\theta_2}{\eta'^2_3-2\eta'_3\{(y_2-D)\cos\theta_2+(y_3-d_2)\sin\theta_2\}+(y_2-D)^2+(y_3-d_2)^2} + \frac{(y_2-D)\sin\theta_2+(y_3-d_2)\cos\theta_2}{\eta'^2_3-2\eta'_3\{(y_2-D)\cos\theta_2-(y_3-d_2)\sin\theta_2\}+(y_2-D)^2+(y_3-d_2)^2}\right]d\eta'_3$$

$$\phi_2(y_2,y_3) = \int_0^{D_2} f_2(\eta'_3)\left[\frac{\eta'^2_3\sin\theta_2-2\eta'_3(y_3-d_2)-\{(y_2-D)^2-(y_3-d_2)^2\}\sin\theta_2+2(y_2-D)(y_3-d_2)\cos\theta_2}{\left[\eta'^2_3-2\eta'_3\{(y_2-D)\cos\theta_2+(y_3-d_2)\sin\theta_2\}+(y_2-D)^2+(y_3-d_2)^2\right]^2} + \frac{\eta'^2_3\sin\theta_2+2\eta'_3(y_3+d_2)-\{(y_2-D)^2-(y_3+d_2)^2\}\sin\theta_2-2(y_2-D)(y_3+d_2)\cos\theta_2}{\left[\eta'^2_3-2\eta'_3\{(y_2-D)\cos\theta_2+(y_3+d_2)\sin\theta_2\}+(y_2-D)^2+(y_3+d_2)^2\right]^2}\right]d\eta'_3$$

$$\phi_3(y_2,y_3) = -\int_0^{D_2} f_2(\eta'_3)\left[\frac{\eta'^2_3\cos\theta_2-2\eta'_3(y_2-D)+\{(y_2-D)^2-(y_3-d_2)^2\}\cos\theta_2+2(y_2-D)(y_3-d_2)\sin\theta_2}{\left[\eta'^2_3-2\eta'_3\{(y_2-D)\cos\theta_2+(y_3-d_2)\sin\theta_2\}+(y_2-D)^2+(y_3-d_2)^2\right]^2} - \frac{\eta'^2_3\cos\theta_2-2\eta'_3(y_2-D)+\{(y_2-D)^2-(y_3+d_2)^2\}\cos\theta_2-2(y_2-D)(y_3+d_2)\sin\theta_2}{\left[\eta'^2_3-2\eta'_3\{(y_2-D)\cos\theta_2+(y_3+d_2)\sin\theta_2\}+(y_2-D)^2+(y_3+d_2)^2\right]^2}\right]d\eta'_3$$

It has been observed, as in [4] that the strains and the stresses will remain bounded everywhere in the model, including the upper and lower edges of the faults, the functions f_1 and f_2 should satisfy the following sufficient conditions:

(I) $f(y_3)$, $f'(y_3)$ are continuous in $0 \le y_3 \le D_1$,

II) Either (a) $f''(y_3)$ is continuous in $0 \le y_3 \le D_1$,

or (b) $f''(y_3)$ is continuous in $0 \le y_3 \le D_1$, except for a finite number of points of finite discontinuity in $0 \le y_3 \le D_1$,

or (c) $f''(y_3)$ is continuous in $0 \le y_3 \le D_1$,except possibly for a finite number of points of finite discontinuity and for the ends points of $(0, D_1)$, there exist real constants m<1 and n<1 such that $y_3{}^m f''(y_3) \to 0$ or to a finite limit as $y_3 \to 0+0$ and $(D_1 - y_3)^n f''(y_3) \to 0$ or to a finite limit as $y_3 \to D_1 - 0$ and

(III) $f(D_1) = 0 = f'(D_1)$, $f'(0) = 0$,

These are sufficient conditions which ensure finite displacements, stresses and strains for all finite (y_2, y_3, t) .

We can evaluate the integrals if $f(y_3)$ is any polynomial satisfying (I),(II) and (III). One such function is

$$\mathrm{f}(y'_3) = \frac{y'^2_3(y'_3-D_1)^2}{\left(\frac{D_1}{2}\right)^4}$$

Similar conditions for the fault F_2.

6. NUMERICAL COMPUTATIONS

We consider $f_1(\xi'_3)$ to be

$$f_1(\xi'_3) = \frac{\xi'^2_3(\xi'_3-D_1)^2}{\left(\frac{D_1}{2}\right)^4}$$

(and a similar function for $f_2(\eta'_3)$) which satisfies all the conditions for bounded strain and stresses stated above.

Following [5], [6] and the recent studies on rheological behaviour of crust and upper mantle by [7], [8] the values to the model parameters are taken as:

$\mu =$ 3.5 x 10^{11}dyne/sq.cm.
η = 5x10^{20} poise
d_1 *and* d_2 =Depth of the fault = 5 km. and 10 km. (noting that the depth of the major earthquake faults are in between 10-30 km.)
$t_1 = t - T_1$
$t_2 = t - T_2$
$\tau_\infty(t) = \tau_\infty(0)(1 + kt)$, $k = 10^{-9}$
$\tau_\infty(0) = 50$ bar
$(\tau_{12})_0 = 50$ bar
$(\tau_{13})_0 = 50$ bar
$(\tau_c)_1 = 200$ bar
$(\tau_c)_2 = 250$ bar
D = 10 km. = Distance measure along the horizontal axes between the upper edges of the fault.
D_1= 8 km.
D_2= 10 km.

We compute the following quantities:

1.Surface share strain e_{12} due to the tectonic forces against time given by

$$E_{12} = e_{12} - (e_{12})_0 = \tau_\infty(0)\left[\frac{kt}{\mu}+\frac{t}{\eta}+\frac{kt^2}{2\eta}\right] + H(t-T_1)\frac{v_1t_1}{2\pi}\psi_2(y_2,y_3) + H(t-T_2)\frac{v_2t_2}{2\pi}\phi_2(y_2,y_3)$$

2. Share stress across the fault against depth along a line L: $y_2 = 5$ km for different inclination of the fault

i) $\theta_1 = \theta_2 = 60^0$
ii) $\theta_1 = \theta_2 = 90^0$

3. i) Shear stress induced by the creeping movement across F_1 at different points near the fault F_2

ii) Shear stress induced by the creeping movement across F_2 at different points near the fault F_1

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4. Total stress at a point y_2=15 km, y_3=15 km against time for different creeping velocities $v_1$and v_2.

5. Region of stress accumulation and release due to the fault movement
i) across F_1 only
ii) across F_2 only
iii) across F_1 and F_2
and their contour representation.

7. DISCUSSION OF THE RESULTS

1. Fig. (3) shows the surface share strain increases with time at increasingly higher rate. The magnitude is of the order of 10^{-3}. This because of the fact that the stress generated due to the tectonic forces is increasing with time. This has been found to be quite reasonable in view of the observational results.

2. Fig. (4) shows the variation of $\tau_{1'2'}$ against depth along a line L for which y_2= 5 km. It is found that there is a region of stress accumulation followed by a region of stress release and accumulation successively. At a depth of about 20 km the magnitude of the stresses becomes negligibly small. The nature of the dependence of the stress with depth depend upon the inclination of the fault quantitively but they have similar quantitive characters.

3. i) Fig. (5a) shows the shear stress generated due to the creeping movement across F_1 in the vicinity of the fault F_2. It is found that the magnitude of the stress near F_2 increases gradually as we move along F_2 from its upper edge and attains a maximum just below its middle point. There after the stress remaining almost the same with a decreasing trends. For a first few kilometres from the upper edge there is a region of stress release followed by a region of stress accumulation.

ii) However the scenario near the fault F_1 (Fig. 5b) due to the creeping movement across F_2 is totaly different. The entire region falls under the region of stress release. The magnitude of the stress release is found to increase as we move downward along the fault F_1 reaching a point of maximum release near the middle of the fault.

4. Fig. (6) shows the stress $\tau_{1''2''}$ against time at a point y_2=15 km, y_3=15 km for different creep velocities given by $v_1 = v_2 = 0$ cm/year, 10 cm/year, 15 cm/year and 20 cm/year. The stress is found to be gradually increasing up to a time $T_1 = 117$ year just prior to the first fault movement. There after the rate of accumulation of stress is found to increase with increasing velocities of creep. This continues up to $t = T_2 = 152$ years (prior the second fault movement). Due to the creeping movement across F_2 the magnitude of the stress falls down, heigher the creep velocity across F_2 more the stress drop. However, due the increasing $\tau_\infty(t)$ the reduced stresses starts increasing once again make up the stress drop a few decades. It may be noted that due to the movement across F_1, the stress at the selected point increases,while for movement across F_2, the said stress decreases at that point. This can be verified from Fig. 5(a) and 5(b).

5.i) Fig. (7a) shows the region of stress accumulation/release due to the fault movement across F_1 only at $t = T_1 + 1$ year.

ii) Fig. (7b) shows the region of stress accumulation/release due to the fault movement across F_2 only at $t = T_2 + 1$ year.

iii) Fig. (7c) shows the region of stress accumulation/release due to the fault movement across both F_1 and F_2 one year after T_2.

6. The region of stress accumulation/release depicted in the above figure has been shown by using stress contour map in Fig. (8a), (8b) and (8c).

8. ACKNOWLEDGEMENTS

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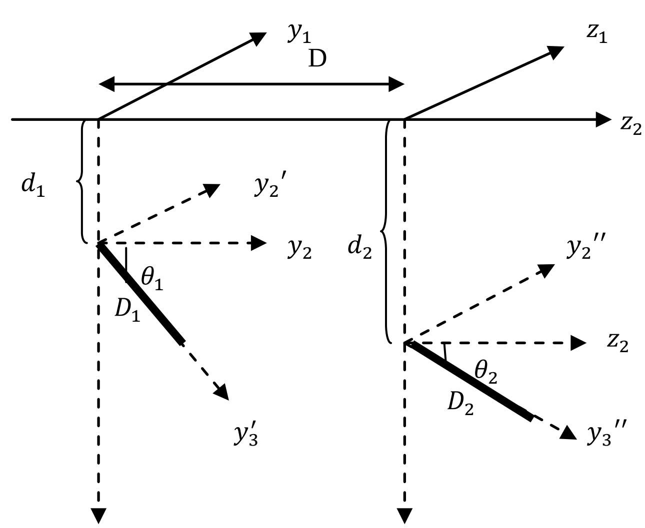

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y_3 z_3

Figure. 1. The section of the fault system by the plane $y_1 = 0$ and relevant coordinate axes

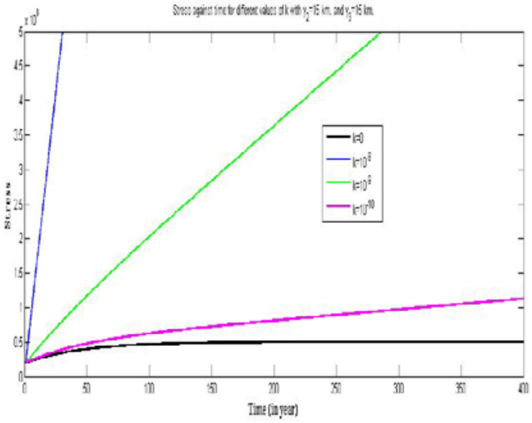


Figure 2. Stress $\tau_{1'2'}$ (before fault movement) against time for different values of k

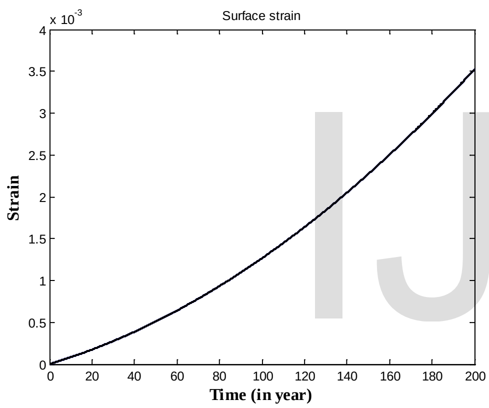


Figure 3. Surface share strain before the fault movement

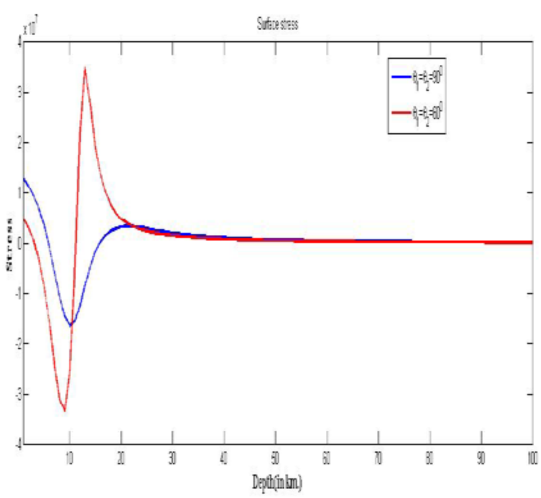


Figure 4. Stress against depth

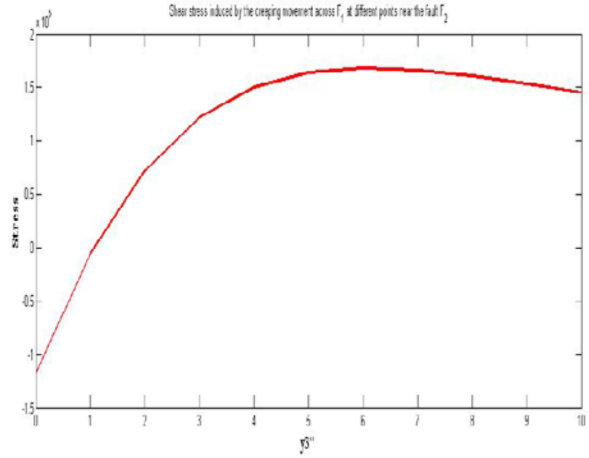


Figure 5a. Shear stress induced by the creeping movement across F_1 at different points near the fault F_2

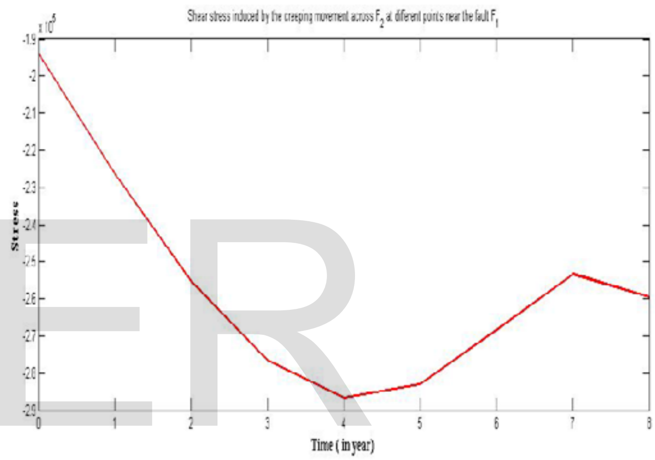


Figure 5b. Shear stress induced by the creeping movement across F_2 at different points near the fault F_1

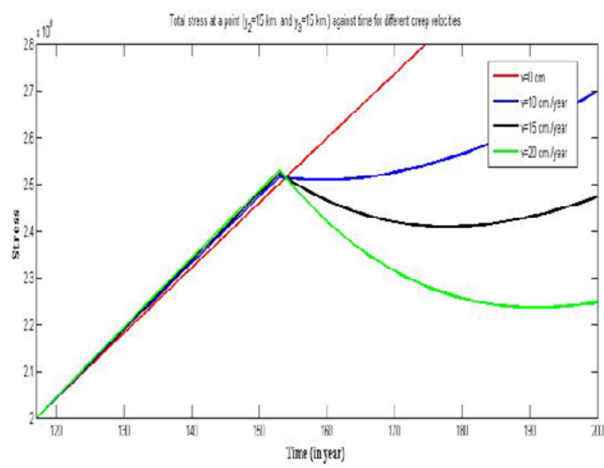
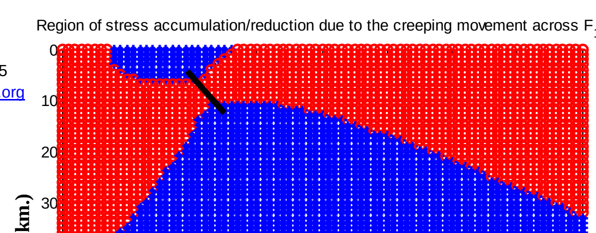


Figure 6. Stress near the mid point on the fault (y_2 =15 km. and (y_3 =15 km.) against time for different creep velocities



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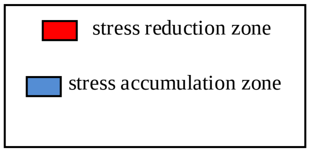

Figure 7a. Region of stress accumulation/reduction due to the creeping movement across F_1

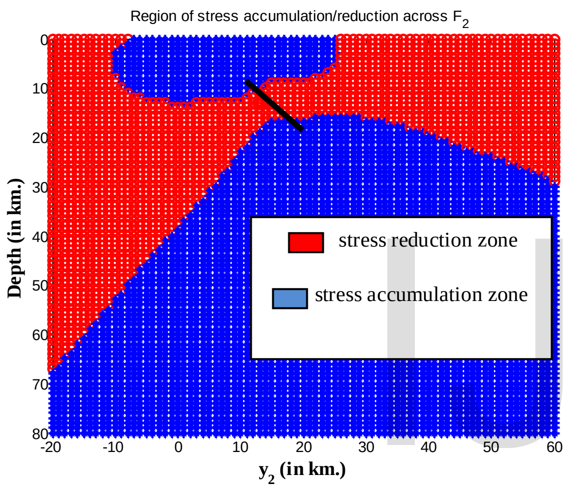

Figure 7b. Region of stress accumulation/reduction due to the creeping movement across F_2

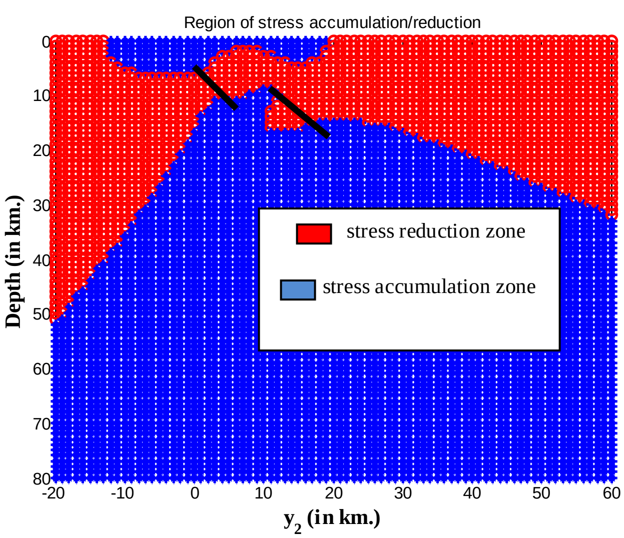

Figure 7c. Region of stress accumulation/reduction

Figure 8a. Contour plot of shear stress due to the creeping movement across F_1

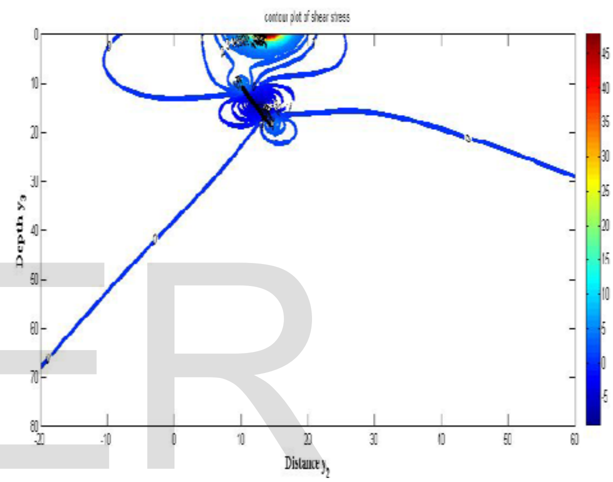

Figure 8b. Contour plot of shear stress due to the creeping movement across F_2

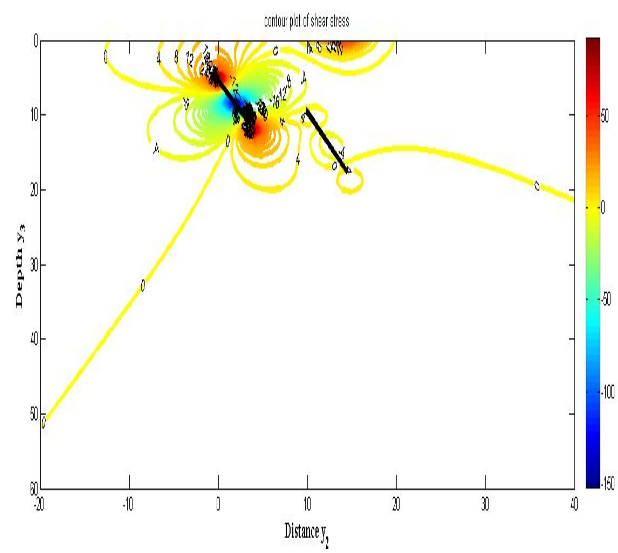

Figure 8c. Contour plot of shear stress

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